UNIVERSITÄT DES SAARLANDES
FR 6.2 – Informatik
Christoph Weidenbach

Lecture "Automated Reasoning"
(Summer Term 2008)

Supplementary Examination

Name: ....

Student Number: ....

Some notes:

- Things to do at the beginning:

  Put your student card and identity card (or passport) on the table.
  Switch off mobile phones.
  Whenever you use a new sheet of paper (including scratch paper), first write your name and student number on it.

- Things to do at the end:

  Mark every problem that you have solved in the table below.
  Stay at your seat and wait until a supervisor staples and takes your examination text.
  Note: Sheets that are accidentally taken out of the lecture room are invalid.

Sign here: **Good luck!**

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Problem	1	2	3a	3b	4	5	6	7	Σ
Answered?									
Points									

Problem 1 *(DPLL(LA))* (10 points)

Refute the 4 clauses

$$x < 5 \vee y \leq 6,\ x' \leq x + 1,\ x' > 6,\ y > 2x'$$

via DPLL(LA) using the Fourier-Motzkin procedure.

Problem 2 *(Miniscoping)* (10 points)

Transform the following formula into clause normal form using miniscoping,

$$\neg[\exists x\, \forall y\, \exists z\, (R(x,x) \vee (P(y) \wedge R(x,y) \wedge Q(z)))]$$

i.e., generate a negation normal form, apply miniscoping, do variable renaming, apply standard Skolemization, and then transform the resulting formula into clause normal form.

Problem 3 *(Orderings)* (6 + 6 = 12 points)

As usual, x, y, z denote variables and a, b constants.

Part (a) For the following term pairs, find if possible a precedence for the LPO such that the left term gets larger than the right term. If the terms cannot be ordered using the LPO, please provide a justification.

- $h(a),\ f(g(b), h(b))$
- $f(f(x,y), h(z)),\ f(g(y), f(x,z))$
- $g(h(x)),\ g(f(x,x))$

Part (b) For the following term pairs, find if possible a precedence and weighting function for the KBO such that the left term gets larger than the right term. If the terms cannot be ordered using the KBO, please provide a justification.

- $h(a),\ f(g(b), h(b))$
- $f(g(x), g(y)),\ g(f(h(y), h(x)))$
- $h(g(f(x,x))),\ g(f(h(x), h(x)))$

Problem 4 *(Superposition)* (16 points)

For the following given superposition rule and premise(s), determine the maximal literal(s) using an LPO with precedence $f > g > h > a$ and compute one conclusion if the rule is applicable. If the rule is not applicable at all, justify why. Check ordering restrictions a priori (before application of the unifier). No selection. No self inferences.

- Positive Superposition:
  $h(x) \approx g(x) \lor h(h(x)) \not\approx x \qquad f(x,y) \approx h(a) \lor h(y) \approx f(g(a), f(x,y))$
- Negative Superposition:
  $f(x,y) \approx h(y) \lor f(x,y) \not\approx y \qquad f(x,y) \approx h(x) \lor h(x) \approx g(x)$
- Equality Resolution:
  $f(y,x) \not\approx f(g(y), g(x)) \lor f(h(x), y) \not\approx h(y)$
- Equality Factoring:
  $h(f(x,z)) \approx g(x) \lor f(g(z), y) \approx h(x) \lor f(x,z) \approx z$

Problem 5 *(Rewriting)* (10 points)

Consider a clause set $N \cup \{h(x) \approx x\}$. Show that N can be effectively transformed into a clause set N' using the clause $h(x) \approx x$ such that N' does not contain the function symbol h, and $N \cup \{h(x) \approx x\}$ is satisfiable iff N' is satisfiable.

Problem 6 *(Model Construction)* (12 points)

Consider an LPO with precedence $f > g > h > a > b$ and compute R_∞ for the following ground clause set. Determine the maximal terms, literal(s) of the clauses, put the clauses in ascending order and finally compute R_∞.

$$f(a,b) \approx h(a)$$
$$h(f(a,b)) \approx h(b) \ \lor \ f(a,b) \approx b$$
$$f(a,b) \not\approx a \ \lor \ f(h(a),b) \approx b$$
$$a \not\approx h(a)$$
$$g(b) \approx h(a) \lor g(a) \approx h(g(a))$$

Problem 7 *(Semantics)* (10 points)

Prove that the disequation $f(f(f(a))) \not\approx f(a)$ is false in any $\Sigma-$algebra $\mathcal{A}$ with $|U_{\mathcal{A}}| \leq 2$.